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Large-scale geometry of Lie groups

I am a 2$^{\text{nd}}$ year postdoc in Fribourg University (Switzerland), with E. Le Donne. The following describes briefly my research projects, so far and in progress, around the large-scale geometry of Lie groups.

Quasiisometries have been an important theme in the modern development of geometric group theory. In my work, I am primarily interested in quasiisometries of (or between) connected Lie groups equipped with proper geodesic distances.

Many quasi-isometric invariants for such a group G are known and studied [4]. They include the asymptotic cones $\mathrm{Cone}(G)$, that are informally "pictures of the group as seen from the infinity", with all the information coming along, e.g. $\pi_1(\mathrm{Cone}(G))$, but also the growth and the filling invariants, among which the Dehn function δ_G, which measures the difficulty to fill loops of given length in the group in an asymptotic way. Through what they retain on the large-scale geometry of groups, those invariants are sometimes related; for instance if a Lie group G has simply connected asymptotic cones, then δ_G is bounded by a polynomial function. Further, if asymptotic cones are additionaly locally compact then one can bound from above the degree of growth of δ_G [3], and even estimate exactly the growth [9], from the knowledge of a single asymptotic cone.

Quasiisometries are not the only maps that preserve all the features of asymptotic cones, though: so do sublinear bilipschitz equivalences (SBE) [1]. In short, sublinear bilipschitz equivalence are obtained by replacing the additive bounds of quasiisometry by a sublinear function of the distance to basepoint. These equivalences occur quite naturally between pairs of nonisomorphic Lie groups provided that these have sufficiently close algebraic structure; they preserve some coarse structures, though rather unusual ones [2,8].

The classification of Lie groups up to sublinear bilipschitz equivalence is necessarily less fine than what we know or expect from the QI classification; nevertheless some invariants can be derived from quasiconformal analysis (in a generalized sense) on the boundaries of Gromov-hyperbolic Lie groups [6,7]. With such techniques, some partial progress can be expected towards the classifications of hyperbolic Lie groups up to QI and SBE, as well as an improved understanding of the large-scale geometry of such groups. On the polynomial growth side, in work triggered by this circle of ideas, following Cornulier and in joint work with C. Llosa Isenrich and R. Tessera, we exhibited pairs of (nilpotent) Lie groups that have biLipschitz simply connected locally compact asymptotic cones, but different Dehn functions [5].

I currently follow two research projects. The first is a collaboration with E. Le Donne and X. Xie, in which we attempt to characterize the connected Lie groups where all te left-invariant proper geodesic distances are roughly similar. The second is a work in progress with Y. Qing, in which we attempt to compare sublinear bilipschitz equivalences and sublinear Morse boundaries, introduced in [10].

Bibliography

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